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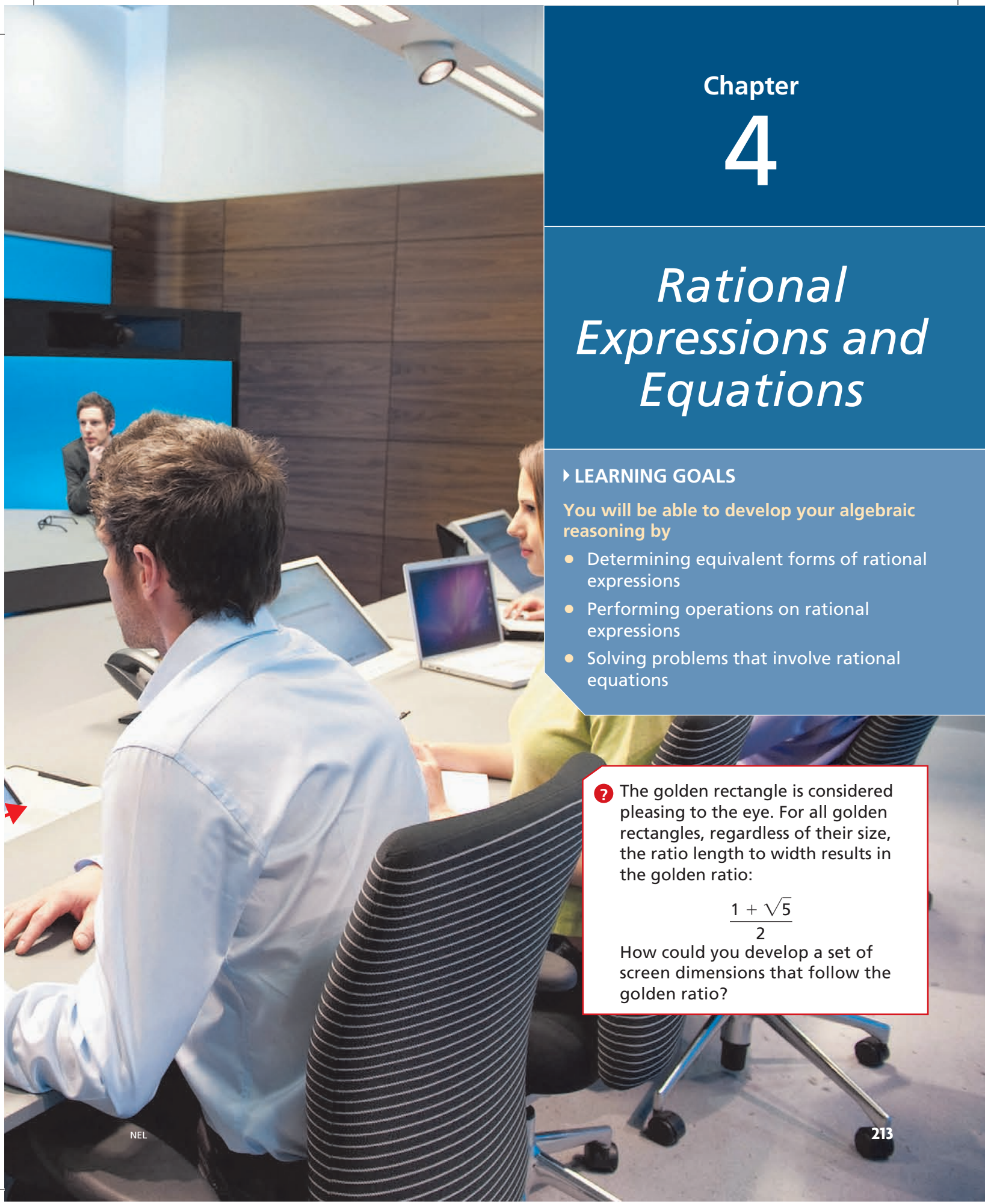

Chapter

4

Rational Expressions and Equations

LEARNING GOALS

You will be able to develop your algebraic reasoning by

- Determining equivalent forms of rational expressions
- Performing operations on rational expressions
- Solving problems that involve rational equations

? The golden rectangle is considered pleasing to the eye. For all golden rectangles, regardless of their size, the ratio length to width results in the golden ratio:

$$\frac{1+\sqrt{5}}{2}$$

How could you develop a set of screen dimensions that follow the golden ratio?

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4 Getting Started

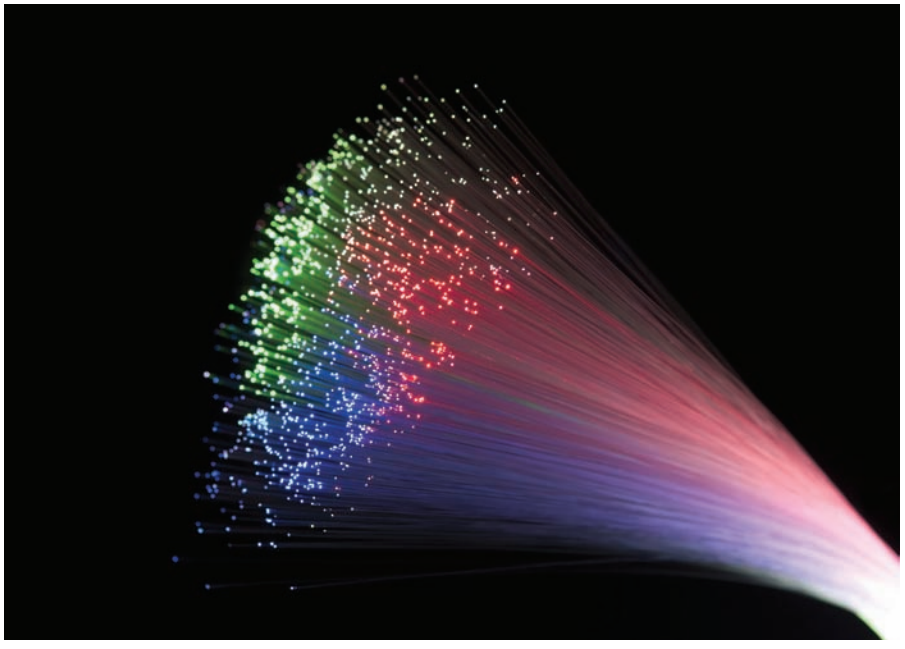
YOU WILL NEED

- calculator

Comparing Internet Plans

In the early days of home Internet access, around 1993, most people connected to the Internet through their telephone line, using a dial-up modem. Modem speeds initially ranged from 14.4 kbps (kilobits per second) to 28.8 kbps, and eventually reached 56 kbps. At the time, 56 kbps was impressive, but users still spent more time downloading content than enjoying it.

In 1996, broadband Internet was introduced to homes in Canada. Users no longer had to wait while their computer dialled up an access number, so connection speeds were five times faster. As well, broadband Internet allowed users access to video content, live voice streaming, and much more. At today's broadband speeds, users can stream high-definition live video and interact globally like never before.



Fibre-optic cable is used in telecommunications because it allows data to be transmitted at high speeds. Laboratory speeds as high as 100 Pb (petabits), or 100 000 000 Gb (gigabits), per second have been recorded.

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Shannon is trying to decide on the right Internet plan for her home (2011 prices). She currently has the least expensive plan, with a download speed of 1 Mbps (megabit per second). She would like to have a connection speed that is fast enough to download movies, so she compared the times for downloading a movie of 4 GB (gigabytes).

Internet Plan	Download Speed (Mbps)	Cost ($/month)
current	1.0	17
A	7.5	19
B	15.0	25
C	50.0	90
D	100.0	150

? If Shannon upgrades to a new high-speed Internet plan, how much time will she save when downloading a 4 GB movie?

A. Determine how long it takes for Shannon to download a 4 GB movie with her current plan.

B. Write an expression for the time it takes to download 4 GB when the download speed is unknown.

C. Are there any restrictions on the expression you wrote in part B? Explain.

D. Write an expression for the difference in download times.

E. Use the expression you wrote in part D to determine the time that Shannon will save with each Internet plan.

Communication *Tip*

One byte (B) is equivalent to 8 bits (b):

1 B = 8 b

One megabyte (MB) is equivalent to 8 megabits (Mb):

1 MB = 8 Mb
1 MB = 1 000 000 B
1 GB = 1000 MB

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. Operations with numbers are identical to operations with algebraic expressions.

2. A variable in an algebraic expression may take any value.

3. When you simplify an algebraic expression, the result is always equivalent to the expression you started with.

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4.1 Equivalent Rational Expressions

GOAL

Determine if two rational expressions are equivalent, and determine any non-permissible values.

EXPLORE...

- The value of a shipment of identical televisions is $45 000. The number of televisions is unknown. Determine a range of possible costs for each television.

LEARN ABOUT the Math

Josephine is an accomplished long-distance runner on her school track team. She has completed 3 km of a 10 km race in 15 min. She wants to finish the race with an average speed of 0.25 km/min.

? In how many minutes must Josephine run the remaining 7 km?

EXAMPLE 1 Writing a fraction equivalent to a rational expression

Determine the time, in minutes, that Josephine must run the remaining 7 km.

Andrew's Solution

Let x represent the time needed for Josephine to finish the race.

Josephine's average speed for the race is total distance divided by total time.

$$\frac{10}{15 + x} \text{ km/min}$$

By analyzing the unit km/min, I know that the formula for speed is

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The total distance is 10 km, and the total time is the original 15 min plus the remaining time, x. This gave me a **rational expression** in x for Josephine's average speed.

rational expression

A fraction that has a polynomial for both the numerator and the denominator; for example:

$$\frac{2x}{3x + 4}$$

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Josephine is aiming for an average speed of 0.25 km/min, which I expressed as a fraction.

$$\frac{1}{4}\text{ km/min}$$
$$= \frac{1}{4} \cdot \frac{10}{10}\text{ km/min}$$
$$= \frac{10}{40}\text{ km/min}$$

I wrote an equivalent fraction for $\frac{1}{4}$ with 10 in the numerator, so I could compare the denominators of the two rational expressions.

$$\frac{10\text{ km}}{15 + x\text{ min}} = \frac{10\text{ km}}{40\text{ min}}$$
$$15 + x = 40$$
$$x = 25$$

I compared the fraction representing Josephine's desired average speed to the rational expression representing her actual average speed. The only way the two rational expressions can be equivalent if the numerators are equal, is if the denominators are also equal.

Josephine needs to finish the remaining 7 km in 25 min if she wants to run the race at an average speed of 0.25 km/min.

Reflecting

A. How could you solve this problem by writing a rational expression that is equivalent to $\frac{10}{15 + x}$? Explain.

B. Determine the **non-permissible values** of x for the expression $\frac{10}{15 + x}$. Explain why these values are non-permissible.

C. Are there any values of x are not permitted in this context? Explain.

non-permissible value

The value of a variable that makes the denominator of a rational expression equal to zero.

For example, $\frac{10}{15 + x}$ is undefined when $x = -15$, because this creates a zero denominator. Therefore, $x = -15$ is a non-permissible value for $\frac{10}{15 + x}$.

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APPLY the Math

EXAMPLE 2 Determining equivalent rational expressions

a) Write a rational number that is equivalent to $\frac{8}{12}$.

b) Write a rational expression that is equivalent to $\frac{4x^2 + 8x}{4x}$.

Nathan's Solution: Determining an equivalent expression using multiplication

a) $\frac{8}{12} = \frac{8}{12} \cdot \frac{5}{5}$

To determine an equivalent rational number, I multiplied the number by a fraction equivalent to 1. Doing this does not change the value of the rational number.

$= \frac{40}{60}$

This is an equivalent rational number.

b) $\frac{4x^2 + 8x}{4x}, x \neq 0$

I knew that this is a rational expression because there is a polynomial in the numerator and the denominator.

Since x is in the denominator, I needed to state any restrictions on x to ensure that the expression is always defined.

$\frac{4x^3 + 8x^2}{4x^2} = \frac{4x^2 + 8x}{4x} \cdot \frac{x}{x}$

Rational expressions are like fractions. To create an equivalent rational expression, I can use the same strategy I used for the rational number. I multiplied the rational expression by $\frac{x}{x}$. I did not introduce a factor that required a new restriction, because the expressions would not have been equivalent.

There is a binomial in the numerator, so the x distributes onto each term.

$= \frac{4x^3 + 8x^2}{4x^2}, x \neq 0$

This is an equivalent rational expression.

Sandra's Solution: Determining an equivalent expression using division

a) $\frac{8}{12} = \frac{4(2)}{4(3)}$

I can write equivalent fractions by dividing the numerator and the denominator by a common factor when one exists.

I simplified the expression by dividing both the numerator and denominator by 4.

$= \frac{2}{3}$

This is an equivalent rational number.

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b) $\frac{4x^2 + 8x}{4x}, x \neq 0$

Since x is in the denominator, I needed to state a restriction to avoid dividing by zero.

$$\frac{4x^2 + 8x}{4x} = \frac{4x(x + 2)}{4x}$$

To determine a second equivalent rational expression, I simplified the factored rational expression by dividing the numerator and denominator by a common factor, $4x$.

$$= x + 2, x \neq 0$$

This is an equivalent rational expression. I needed to keep the restriction because two rational expressions are equivalent only if they have the same restrictions. Equivalent expressions are equal when they are evaluated for any permitted value of the variable.

Your Turn

Verify that the rational expression in one of the solutions above is equivalent to the given expression by evaluating both expressions for a chosen value of x. Explain what happens if your chosen value is the non-permissible value.

EXAMPLE 3 Determining the non-permissible values for a rational expression

Determine the non-permissible value(s) for each rational expression, and then state all restrictions.

a) $\frac{4x^3}{4 - x}$ **b)** $\frac{-15}{x^2 - 5x}$

Naomi's Solution: Determining when the denominator is zero

a) $\frac{4x^3}{4 - x}$

The denominator of this rational expression is $4 - x$.

Determine the non-permissible values:

$4 - x = 0$

$x = 4$

The non-permissible values occur when $4 - x$ equals 0.

There is one non-permissible value, $x = 4$.

$\frac{4x^3}{4 - x}, x \neq 4$

I wrote the expression with the restriction to ensure that the expression is defined.

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b) $\dfrac{-15}{x^2 - 5x}$

The denominator of this rational expression is $x^2 - 5x$.

Determine the non-permissible values:

$x^2 - 5x = 0$

$x(x - 5) = 0$

$x = 0$ and $x - 5 = 0$

$x = 5$

To determine the non-permissible values, I set the denominator equal to 0. I factored the quadratic equation and analyzed each factor.

There are two non-permissible values, $x = 0$ and $x = 5$.

$\dfrac{-15}{x^2 - 5x}, \quad x \neq 0, x \neq 5$

I wrote the expression with the restrictions to ensure that the expression is defined.

Your Turn

Write a rational expression that has non-permissible values of $x = -2$ and 0.
State the restrictions on your expression.

EXAMPLE 4 Determining if rational expressions are equivalent

For each of the following, determine if the rational expressions are equivalent.

a) $\dfrac{9}{3x - 1}$ and $\dfrac{-18}{2 - 6x}$ **b)** $\dfrac{2 - 2x}{4x}$ and $\dfrac{x - 1}{2x}$

Hannah's Solution

a) Determine the non-permissible values:

$\dfrac{9}{3x - 1}$

$3x - 1 = 0$

$3x = 1$

$x = \dfrac{1}{3}$

$\dfrac{-18}{2(1 - 3x)}$

$1 - 3x = 0$

$1 = 3x$

$\dfrac{1}{3} = x$

I factored the denominator of the second expression then set each factor of the denominator that involved a variable equal to 0.

The denominators of both expressions have the same non-permissible value, $x = \dfrac{1}{3}$.

Compare the rational expressions:

$\dfrac{9}{3x - 1}$

$= \dfrac{9}{3x - 1} \cdot \dfrac{-2}{-2}, x \neq \dfrac{1}{3}$

$= \dfrac{-18}{-6x + 2}$

$\dfrac{-18}{2(1 - 3x)}, x \neq \dfrac{1}{3}$

$= \dfrac{-18}{2 - 6x}$

The numerators of the two expressions differ by a factor of -2.

I multiplied the numerator and the denominator of the first rational expression by -2 to determine if the first and second expressions are equivalent.

Since $2 - 6x$ can be written as $-6x + 2$, I can see that both denominators are the same.

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$$\frac{9}{3x-1} = \frac{-18x}{2-6x}, \quad x \neq \frac{1}{3}$$

The two rational expressions are equivalent for all permissible values.

b) Determine the non-permissible values:

$\frac{2-2x}{4x}, \ x \neq 0$ | $\frac{x-1}{2x}, \ x \neq 0$

The value that makes the denominators equal to zero is the same, $x = 0$.

Use substitution to check equality:

$$= \frac{2-2(2)}{4(2)} \quad \Big| \quad = \frac{2-1}{2(2)}$$
$$= \frac{2-4}{8} \quad \Big| \quad = \frac{1}{4}$$
$$= -\frac{2}{8} \quad \Big|$$
$$= -\frac{1}{4} \quad \Big|$$

I chose a permissible value, $x = 2$. I substituted the values into each expression as a quick check to determine if the expressions might be equal.

The results are not equal.

$$\frac{2-2x}{4x} \neq \frac{x-1}{2x}, x \neq 0$$

Since the rational expressions are not equal for $x = 2$, they are not equivalent.

For rational expressions to be equivalent, they must be equal for all permissible values of the variable.

Your Turn

Max decided that the rational expressions $\frac{x^2-1}{4x}$ and $\frac{5-5x}{3x}$ are equivalent by evaluating each expression for $x = 1$. Is Max's solution correct? Explain.

Max's Solution

$$\frac{x^2-1}{4x} \quad \Big| \quad \frac{5-5x}{3x}$$
$$= \frac{(1)^2-1}{4(1)} \quad \Big| \quad = \frac{5-5(1)}{3(1)}$$
$$= 0 \quad \Big| \quad = 0$$

I substituted 1 for x in each expression.

The two expressions are equivalent.

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In Summary

Key Ideas

- A rational number is the ratio of two integers. A rational expression is the ratio of two polynomials. For example, $-\frac{2}{5}$ is a rational number, and $\frac{5x + 2}{x - 3}$, $x \neq 3$, is a rational expression.
- The strategies that are used to write equivalent forms of a rational expression are similar to those that are used to write equivalent forms of a rational number. These strategies include
  - multiplying the numerator and the denominator by the same factor, and
  - dividing the numerator and the denominator by a common factor.

Need to Know

- A rational number is undefined when the denominator is equal to zero. As a result, all rational expressions are undefined for values of a variable that make the denominator equal to zero. These values of the variable are called non-permissible values.
- The non-permissible values of the variables in a rational expression are the values that make the denominator equal to zero. If the denominator contains a binomial, the non-permissible values can be determined from the factored form of the binomial.
- All non-permissible values of a rational expression must be stated as restrictions on the variable to ensure that the expression is defined.
- Substitution can be used to determine if two rational expressions are *not* equivalent, but it cannot be used to determine if two rational expressions are equivalent.

CHECK Your Understanding

1. a) Write an equivalent rational expression for each expression below.

i) $\frac{16x^3}{10x}$ **ii)** $\frac{2x + 6}{4}$ **iii)** $\frac{x - 2}{4x^2}$ **iv)** $\frac{x - 3}{x - 1}$

b) State the non-permissible values for each pair of equivalent expressions.

c) Choose one pair of equivalent expressions from part a). Demonstrate that they are equivalent by substituting a permissible value into both expressions.

2. Define a variable and write a rational expression to describe each situation below. Remember to state the non-permissible values.

a) The speed of a train that is travelling 450 km

b) The average rate of a glacier melt that has been observed over 15 years

The Athabasca Glacier in Jasper National Park, Alberta, has lost half its volume over the past 125 years.

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PRACTISING

3. Determine the non-permissible values of the variable in each rational expression below, and write two equivalent rational expressions.

a) $\frac{2 - 7x}{-5 - x}$ **b)** $\frac{4a^2}{a^2 + a}$ **c)** $\frac{4t^3}{3t^2 - t}$ **d)** $\frac{-2x}{4x - 8}$

4. Identify the equivalent rational expressions. Explain why they are equivalent.

A. $\frac{4a - 4}{16a^2}$ **B.** $\frac{2 - 2a}{32a^2}$ **C.** $\frac{8a - 8}{-128a^2}$ **D.** $\frac{16a^2 - 16a}{64a^3}$

5. Identify the rational expressions that are equivalent to $\frac{48x^2}{16x - 4}$. Explain why they are equivalent.

A. $\frac{12x^2}{4x - 1}$ **B.** $\frac{24x^2}{8x - 4}$ **C.** $\frac{3x^2}{x - 4}$ **D.** $\frac{36x^2}{48x - 12}$

6. One hose can fill a swimming pool in y hours. A larger hose can fill the same pool 20 min faster.

a) Write a rational expression for the fraction of the pool that is filled by the larger hose in 1 h.

b) What are the restrictions on the variables? Explain.

7. The resistance of blood within your vascular system can be an important indicator for your risk of having a heart attack or stroke. Poiseuille's law gives an expression for the resistance of blood, based on the length, L, and the radius, r, of a blood vessel. For a blood vessel that is 2 cm long, Poiseuille's law states that the resistance is $\frac{36}{r^4}$. Determine any non-permissible values for this expression, and explain why they are non-permissible.

High blood pressure affects one in five Canadians. High blood pressure is an important risk factor for stroke and heart disease.

8. A fish can swim at a speed of v km/h in still water. An expression for the time it takes the fish to swim 3 km upstream in a river that is moving at 5 km/h is $\frac{3}{v - 5}$. Determine any non-permissible values for the expression, and explain why they are non-permissible.

9. **a)** State the non-permissible value(s) for each rational expression.

i) $\frac{2x - 1}{x + 5}$ **ii)** $\frac{-6x + 3}{-3x - 15}$ **iii)** $\frac{4x^3 - 2x^2}{2x^3 + 10x^2}$

b) Are these rational expressions equivalent? Explain how you know.

10. Write a rational expression that has the given restriction(s) as its non-permissible value(s).

a) $x \neq 0$ **b)** $a \neq 0, 3$ **c)** $x \neq -5, 0$ **d)** $a \neq \frac{1}{3}$

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11. **a)** Determine the non-permissible values for each rational expression.

i) $\frac{a + 2}{3a - 2}$ **ii)** $\frac{4x^4}{16x^2 - 2x}$ **iii)** $\frac{-15 - 6y}{3y^3}$

b) Write two equivalent rational expressions for each expression in part a).

12. Explain how determining an equivalent rational expression is similar to determining an equivalent rational number. Use examples in your explanation.

13. Explain how determining an equivalent rational expression is different from determining an equivalent rational number. Use examples in your explanation.

14. Use multiplication to write a rational expression with a non-permissible value that is equivalent to $\frac{2x - 7}{3x}$. State the non-permissible value.

15. Use division to write a rational expression that is equivalent to $\frac{4x^3 - 5x}{12x}$. State the non-permissible value(s).

16. State the non-permissible values for each rational expression below.

a) $\frac{5a^4}{5a^2 - 7a}$ **b)** $\frac{-1}{15x^2 + 10x}$ **c)** $\frac{3x^5}{x^2 + 9}$ **d)** $\frac{2a - 2}{4a^2 - 4}$

Closing

17. **a)** Create a rational expression with two non-permissible values.

b) Exchange rational expressions with a classmate. Determine two rational expressions that are equivalent to your classmate's expression, and state the non-permissible values.

Extending

18. The following expression is an example of a six-term geometric series:

$$2 + 2r + 2r^2 + 2r^3 + 2r^4 + 2r^5$$

It is called a "geometric" series because the ratio of the consecutive terms is constant. For this series, the constant ratio is r.

When $r = 1$, the sum of the series is 12. When $r \neq 1$, the rational expression that represents the sum of this series is:

$$\frac{2(1 - r^6)}{1 - r}$$

a) Demonstrate that the rational expression is accurate by choosing a value for r.

b) What do you think the rational expression would look like if this series had 10 terms? Verify that your prediction works.

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4.2 Simplifying Rational Expressions

GOAL

Simplify rational expressions.

INVESTIGATE the Math

The cost of advertising space in a local newspaper is based on the area of the advertising space. One customer has paid for an advertisement that has an area of $15y$ cm^2, where y is the length of the space. The newspaper editor needs to find a location in the paper where the advertisement will fit.

? What is the simplified rational expression for the width of the rectangular advertising space?

A. Use the formula for the area of a rectangle to write an expression for the width of a rectangle.

B. Substitute the area and length of the advertising space into your expression for the width of a rectangle. Is this a rational expression? Explain.

C. Are there any non-permissible values for the expression determined in part B? Give two reasons why the values are non-permissible.

D. Write a simplified equivalent rational expression for the expression in part B. Do your non-permissible values still make sense for the problem? Explain.

EXPLORE...

- Use factoring and elimination to write three rational expressions that are equivalent to $\frac{-12x^3}{18x^2 + 18x^3}$.

Communication *Tip*

- A rational expression is in simplified form when the numerator and the denominator have no common factors.
- Always state the non-permissible values of the variables as restrictions before simplifying a rational expression. Otherwise, if you eliminate a variable or factor as you simplify, you will lose the information about the non-permissible value of the variable.

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Communication Tip

An inadmissible value for a variable in a rational expression is a value that does not work in the context of the problem. For example, if $\frac{100}{x}$ is a rational expression that represents the length of a rectangle with width x, then $x = 0$ is a non-permissible value and $x < 0$ represents the inadmissible values. It does not make sense to have a rectangle with a negative width.

Reflecting

E. What operation did you use to simplify? Explain.

F. Explain why you need to state the non-permissible values for a rational expression before you simplify the expression.

G. Explain why the non-permissible values remain after the rational expression is simplified.

H. What values of y are **inadmissible** in this problem? Explain.

APPLY the Math

EXAMPLE 1 Simplifying a rational expression with monomials

Simplify the following rational expression:

$$\frac{-24a^2}{18a^3}$$

Teagan's Solution

$\frac{-24a^2}{18a^3}, a \neq 0$

First, I determined the non-permissible values of the variable.

$\frac{-24a^2}{18a^3} = \frac{-4(6a^2)}{3a(6a^2)}$

$= \frac{-4}{3a}$

I factored the numerator and the denominator to identify the common factors. Then I divided the numerator and denominator by the common factor to simplify the rational expression.

$= \frac{-4}{3a}, a \neq 0$

This rational expression is simplified because the numerator and the denominator have no common factors other than 1.

Your Turn

Simplify this expression. Remember to state the non-permissible values of the variable as restrictions before you simplify.

$$\frac{18x^4}{36x^7}$$

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EXAMPLE 2 Simplifying a rational expression with a binomial numerator

Simplify the following rational expression:

$$\frac{15x^3 - 5x}{15x^3}$$

Noel's Solution: Factoring the rational expression

$\frac{15x^3 - 5x}{15x^3}, \quad x \neq 0$

I determined the values that make the denominator equal to zero.

$\frac{15x^3 - 5x}{15x^3} = \frac{5x(3x^2 - 1)}{5x(3x^2)}$

I factored the numerator and the denominator to identify the common factors. Then I divided by the common factor to simplify the rational expression.

$= \frac{3x^2 - 1}{3x^2}, \quad x \neq 0$

I could not simplify the rational expression any further, because the greatest common factor (GCF) of the numerator and the denominator is 1.

Stéphane's Solution: Simplifying a difference of fractions

$\frac{15x^3 - 5x}{15x^3}, \quad x \neq 0$

First, I determined the non-permissible values.

Because this rational expression has a difference in the numerator, I decided to split it into a difference of two rational expressions with a common denominator.

$\frac{15x^3 - 5x}{15x^3} = \frac{15x^3}{15x^3} - \frac{5x}{15x^3}$

$= 1 - \frac{(5x)}{3x^2(5x)}$

$= 1 - \frac{1}{3x^2}$

The first rational expression in the difference is 1.

I factored the second rational expression to identify the common factors. I divided the numerator and denominator of the second rational expression by $5x$ to simplify.

$= 1 - \frac{1}{3x^2}, \quad x \neq 0$

I could not simplify the second rational expression any further, because the GCF of the numerator and the denominator is 1.

Your Turn

Simplify the following expression in two different ways:

$$\frac{3y - 9y^2}{6y^3}$$

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EXAMPLE 3 Simplifying a rational expression with binomials

Simplify the following rational expression:

$$\frac{6m^2 - 8m}{3m^3 - 4m^2}$$

Rachel's Solution

$\frac{6m^2 - 8m}{3m^3 - 4m^2}$

To determine the non-permissible values, I looked for the values that make the denominator equal to zero.

Denominator:
$3m^3 - 4m^2 = 0$

I set the denominator equal to 0 and factored the expression on the left.

$m^2(3m - 4) = 0$
$m^2 = 0$ or $3m - 4 = 0$

I set both factors equal to zero. I solved each equation separately.

$m = 0$ $\qquad 3m = 4$
$\qquad\qquad m = \frac{4}{3}$

$m = 0$ and $m = \frac{4}{3}$ are non-permissible values of the variable in the rational expression.

$\frac{6m^2 - 8m}{3m^3 - 4m^2}, \quad m \neq 0, \frac{4}{3}$

$\frac{6m^2 - 8m}{3m^3 - 4m^2} = \frac{2(m)(3m - 4)}{m(m)(3m - 4)}$

I factored the numerator and the denominator to identify the common factors.

$= \frac{2}{m}, \quad m \neq 0, \frac{4}{3}$

I simplified the rational expression using the fact that $\frac{m}{m} = 1$ and $\frac{3m - 4}{3m - 4} = 1$. I cannot simplify this expression any further because the GCF of the numerator and the denominator is 1.

Your Turn

Simplify the following expression:

$$\frac{3a^3 - 3a^2}{-12a + 12}$$

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In Summary

Key Idea

- A rational expression can be simplified by factoring the numerator and the denominator, and then dividing both by their greatest common factor (GCF).

Need to Know

- A rational expression is in simplified form when the GCF of the numerator and the denominator is 1.
- The non-permissible values must be stated before the rational expression is simplified to ensure all restrictions for the variable are included.

CHECK Your Understanding

1. Which of the following rational expressions are simplified? Explain.

A. $\frac{4x^3}{5x^3 + 5}$ **B.** $\frac{16x^2}{5x}$ **C.** $\frac{3x + 5}{3x}$ **D.** $\frac{3a - 6}{6 - 9a}$

2. Determine the non-permissible values for each rational expression, state these values as restrictions, and simplify.

a) $\frac{12x^4}{14x}$

b) $\frac{4a}{8a^2(a + 3)}$

c) $\frac{2y(2 - y)}{16(y - 2)}$

d) $\frac{(4x + 3)4x^3}{4x^3(3x + 4)}$

PRACTISING

3. Simplify.

a) $\frac{6y}{4y^3}$ **b)** $\frac{-17y^5}{7y^6}$ **c)** $\frac{240y^2}{168y}$ **d)** $\frac{-32a^3}{26a^3}$

4. Simplify.

a) $\frac{20a^2 + 30a^3}{25a^2}$

b) $\frac{6a + 2}{6a}$

c) $\frac{18a^2 - 24a^3}{36a^2}$

d) $\frac{-15x^2 - 10x}{15x^2}$

5. Simplify.

a) $\frac{4x^2 - 18x}{20x - 90}$

b) $\frac{9x^2 - 12x}{6x - 8}$

c) $\frac{10x^2 - 8x}{24x - 30x^2}$

d) $\frac{4x^2 - 8x}{10 - 5x}$

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6. **a)** Simplify: $\dfrac{-10x^2 - 15x}{4x^3 - 9x}$

b) Explain why it is important to determine any non-permissible values of the variable before simplifying.

7. Brad tried to determine the non-permissible values for an expression and then simplify, as shown below.

$\dfrac{4a + 10}{2a} = \dfrac{4a + 10}{2a}, \quad a \neq 0$ — I knew that I couldn't divide by zero, so zero is a non-permissible value of a.

$= 2 + 10$ — $2a$ can be divided into $4a$, and the result is 2.

$= 12, \quad a \neq 0$ — This rational expression cannot be simplified any further.

a) Identify Brad's error, and correct his solution.
b) Explain why Brad's error may be a common error.

8. Sharon tried to simplify an expression, as shown below.

$\dfrac{12x - 6x^2}{10x^3 - 20x^2} = \dfrac{3(2x)(2 - x)}{5x(2x)(x - 2)}$ — I started by factoring the numerator and the denominator.

$= \dfrac{3(2x)(2 - x)}{5x(2x)(x - 2)}$ — I knew that $x - 2$ and $2 - x$ are opposites. When opposites are divided, the result is -1.

$= \dfrac{-3}{5x}, \quad x \neq 0$ — The numerator and the denominator have no more common factors, so the expression cannot be simplified any further. $x = 0$ is a non-permissible value because you cannot divide by zero.

a) Identify Sharon's error, and correct her solution.
b) Explain why Sharon's error may be a common error.

Kitsilano Pool in Vancouver, British Columbia, is a saltwater pool. Its chlorination system uses electrolysis to break down the salt in the water, producing chlorine.

9. Liquid chlorine is added to a swimming pool to sanitize the water. At t hours, the concentration of chlorine in the pool can be expressed as $\dfrac{4t}{4 + 2t}$ mg/L.

a) Determine the concentration of chlorine in the pool at each time.
i) 0 h **ii)** 2 h **iii)** 6 h
b) Simplify the expression for the concentration of chlorine.
c) Does the expression above have any inadmissible values in this context? Explain.

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10. Pruner's Lawn Care specializes in residential lawn cutting and fertilizing. Mike, the owner, has tracked his income and expenses. He has determined that his profit can be represented by the expression $\$4A$, where A is the area of a lawn in square metres. The time, in hours, that it takes him to maintain a lawn is approximately $0.02A + 0.24$.

a) Write a rational expression for Mike's profit per hour.

b) Evaluate the rational expression you wrote in part a) for a yard with an area of 50 m^2.

c) Write a simplified rational expression, without decimals, that is equivalent to the expression you wrote in part a).

d) Are there any inadmissible values for your expression in this context? Explain.

11. Write a rational expression that has a binomial factor in the denominator that gets eliminated when the expression is simplified. State the non-permissible values.

12. Write a rational expression with a binomial in the denominator, so that the denominator is a monomial when the expression is simplified. State the non-permissible values.

13. Simplify.

a) $\dfrac{4x^3 - 36x}{6x^2 - 2x^3}$

b) $\dfrac{121y^3 - y}{y^2 + 11y^3}$

Closing

14. Explain how simplifying a rational expression is similar to writing a fraction in simplest terms. Use an example in your explanation.

Extending

15. The area of rectangle $ABCD$ on a Cartesian grid can be represented by the expression $x^2 + 7x + 10$, where x represents the x-coordinate of a point on the x-axis.

a) Vertex A is placed at the origin, and vertex B is placed at $(x + 2, 0)$. Determine a simplified expression for the y-coordinate of vertex D as a function of x.

b) Use the context of this problem to explain the non-permissible values for the variable.

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4.3 Multiplying and Dividing Rational Expressions

GOAL

Develop strategies for multiplying and dividing rational expressions.

EXPLORE...

- Express $\frac{3}{10}$ as a product of two rational numbers and as a quotient of two rational numbers.

INVESTIGATE the Math

Leanne has noticed that operations with rational expressions are very similar to operations with rational numbers. She decides to investigate multiplication and division of rational expressions to determine whether they are similar to multiplication and division of rational numbers.

? Is the strategy for multiplying rational expressions the same as the strategy for multiplying rational numbers?

A. Explain the strategy for multiplying rational numbers. Use an example in your explanation.

B. Choose any two rational expressions and write them as a product.

$$\frac{5x^2}{2} \qquad \frac{3}{7x} \qquad \frac{-2x^3}{3} \qquad \frac{10}{3x} \qquad \frac{6x}{5x^3}$$

C. Evaluate this product for $x = 2$.

D. Use your strategy for multiplying rational numbers to determine the product of the rational expressions you wrote in part B and simplify the product.

E. Evaluate your simplified expression for $x = 2$. What do you notice?

F. Repeat steps B to E with two other rational expressions from above.

G. Is the strategy for multiplying rational expressions the same as the strategy for multiplying rational numbers? Explain.

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Reflecting

H. Do you think the strategy for dividing rational expressions should be the same as the strategy for dividing rational numbers? How could you check?

I. How are multiplying and dividing rational expressions different from multiplying and dividing rational numbers?

J. How can you determine the non-permissible values of the variable in the product or quotient of two rational expressions?

APPLY the Math

EXAMPLE 1 Simplifying a product

Simplify the following product:

$$\frac{2x^2 - 12x}{15x} \cdot \frac{5x}{x - 6}$$

Liam's Solution

$$\frac{2x^2 - 12x}{15x} \cdot \frac{5x}{x - 6}, \quad x \neq 0, 6$$

$x = 0$ and $x = 6$ make a denominator equal to zero.

$$\frac{2x^2 - 12x}{15x} \cdot \frac{5x}{x - 6} = \frac{2(x)(x - 6)}{15(x)} \cdot \frac{5x}{x - 6}$$

$$= \frac{2(x - 6)}{15} \cdot \frac{5x}{x - 6}$$

$$= \frac{2(x - 6)(5x)}{15(x - 6)}$$

I factored the rational expressions to see if I could simplify before multiplying. I simplified the first rational expression because the numerator and the denominator have a common factor of x.

I multiplied the two rational expressions by multiplying the numerators and multiplying the denominators.

$$= \frac{2(x - 6)(5)x}{3(5)(x - 6)}$$

$$= \frac{2x}{3}, \quad x \neq 0, 6$$

I simplified using common factors. The non-permissible values come from just the denominators in the original product, which is equivalent to this expression.

Your Turn

Simplify the following product:

$$\frac{12x^3}{3x^2 + 6x} \cdot \frac{4x^3 + 8x^2}{5}$$

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EXAMPLE 2 Simplifying a quotient

Simplify each quotient.

a) $\dfrac{x-5}{3x^2-9x} \div \dfrac{5}{6x-18}$

b) $\dfrac{2w}{24w+4w^2} \div \dfrac{6w^2-6w}{9w^3+54w^2}$

Lainey's Solution

a) $\dfrac{x-5}{3x^2-9x} \div \dfrac{5}{6x-18}$

$= \dfrac{x-5}{3x(x-3)} \div \dfrac{5}{6(x-3)}, \quad x \neq 0, 3$

I factored the rational expressions to determine the non-permissible values of the variable. I know that dividing by zero is undefined, so the non-permissible values are $x = 0$ and $x = 3$.

$= \dfrac{x-5}{3x(x-3)} \cdot \dfrac{6(x-3)}{5}$

To divide rational expressions, I multiplied by the reciprocal of the second expression.

$= \dfrac{(x-5)(2)(3)(x-3)}{3x(x-3)(5)}$

I identified the common factors.

$= \dfrac{(x-5)(2)}{5x}, \quad x \neq 0, 3$

I simplified using the fact that $\dfrac{3}{3} = 1$ and $\dfrac{x-3}{x-3} = 1$. The rational expression cannot be simplified any further because the GCF of the numerator and the denominator is 1.

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b) Determine the non-permissible values. Since I am also dividing the first rational expression by the second, I have to make sure that the second rational expression (or divisor) is not equal to zero.

$$\frac{2w}{24w + 4w^2} \div \frac{6w^2 - 6w}{9w^3 + 54w^2}$$
$$= \frac{2w}{4w(6 + w)} \div \frac{6w(w - 1)}{9w^2(w + 6)}, \quad w \neq -6, 0, 1$$

$w = 0$ and $w = -6$ make the denominators equal to zero.

When $w = 1$, the numerator of the second rational expression will equal zero and the second expression itself will be zero when evaluated. This will result in the first rational expression being divided by zero, which is undefined.

Therefore, another non-permissible value is $w = 1$.

$$\frac{2w}{4w(6 + w)} \div \frac{6w(w - 1)}{9w^2(w + 6)} = \frac{(2w)}{2(2w)(6 + w)} \div \frac{2(3w)(w - 1)}{3w(3w)(w + 6)}$$

I used common factors to simplify each rational expression.

$$= \frac{1}{2(6 + w)} \div \frac{2(w - 1)}{3w(w + 6)}$$

To divide, I multiplied by the reciprocal of the second expression, just as I do for rational numbers.

$$= \frac{1}{2(6 + w)} \cdot \frac{3w(w + 6)}{2(w - 1)}$$

I simplified the rational expression.

$$= \frac{1 \cdot 3w(w + 6)}{2(6 + w) \cdot 2(w - 1)}$$

$$= \frac{3w(w + 6)}{4(6 + w)(w - 1)}$$

$$= \frac{3w}{4(w - 1)}, \quad w \neq -6, 0, 1$$

This rational expression is the simplified quotient. The non-permissible values come from the denominator of the first rational expression and the numerator and denominator of the second rational expression in the original quotient, which is equivalent to this expression.

Your Turn

Simplify the following quotient:

$$\frac{x^3 + x^2}{16} \div \frac{x^2 + x}{20x - 10}$$

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EXAMPLE 3 Simplifying a rational expression containing several binomials

Simplify the following expression:

$$\frac{30x^2 + 15x}{x - 3} \div \frac{2x^3 + x^2}{x^2 - 3x}$$

Désirée's Solution

$$\frac{30x^2 + 15x}{x - 3} \div \frac{2x^3 + x^2}{x^2 - 3x}$$

$$= \frac{15x(2x + 1)}{(x - 3)} \div \frac{x^2(2x + 1)}{x(x - 3)}, x \neq -\frac{1}{2}, 0, 3$$

I factored the polynomials. The non-permissible values are $x = 0$ and $x = 3$.

Since the second rational expression will be divided into the first expression, it cannot equal zero. The second rational expression (or divisor) will equal zero when the numerator equals zero. Therefore, $x = -\frac{1}{2}$ is another non-permissible value.

$$= \frac{15x(2x + 1)}{(x - 3)} \cdot \frac{x(x - 3)}{x(x)(2x + 1)}$$

To divide, I multiplied by the reciprocal of the second expression. I identified the common factors in the numerators and denominators to simplify.

$$= \frac{15(2x + 1)}{(x - 3)} \cdot \frac{(x - 3)}{(2x + 1)}$$

To be careful, I did the simplifying over two steps.

$$= 15, x \neq -\frac{1}{2}, 0, 3$$

Your Turn

Simplify the following expression:

$$\frac{4x^2 - 1}{x + 2} \div \frac{4x^2 + 2x}{8x^2 - 32}$$

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In Summary

Key Idea

- The strategies you use to multiply and divide rational numbers can be used to multiply and divide rational expressions. That is, if A, B, C, and D are polynomials, then

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}, B, D \neq 0$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} \text{ or } \frac{AD}{BC}, B, D, \text{ and } C \neq 0$$

Need to Know

- When performing operations with rational expressions, follow these steps:

  To *multiply:*

  1. Factor the numerators and denominators of both expressions, if possible.
  2. Identify the non-permissible values of the variable.
  3. Multiply the numerators and multiply the denominators, writing each product as a single rational expression.
  4. Simplify using common factors.
  5. Write the product, stating the restrictions on the variable.

  To *divide*:

  1. Factor the numerators and denominators, if possible.
  2. Identify the non-permissible values of the variable.
  3. Multiply by the reciprocal of the divisor.
  4. Multiply the numerators, and then multiply the denominators, writing each product as a single rational expression.
  5. Simplify using common factors.
  6. Write the quotient, stating the restrictions on the variable.

- To determine any *non-permissible values* of the variable, determine all the values that make the denominators equal to zero in the factored quotient or product. For division, remember to consider both the numerator and the denominator of the divisor.

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CHECK Your Understanding

1. Determine all the non-permissible values of the variable.

a) $\frac{4x}{(x-3)} \cdot \frac{2x(x+2)}{(3x-1)}$

b) $\frac{8x^3}{(x+4)} \div \frac{x}{(x-1)}$

c) $\frac{4x}{7} \cdot \frac{2x(2x+5)}{12}$

d) $\frac{3}{x-1} \div \frac{x+3}{x}$

2. Simplify.

a) $\frac{4x^2}{27} \cdot \frac{9}{2x^3}$

b) $\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)$

c) $\frac{-6y^2}{18} \div \frac{3y^3}{14}$

d) $\frac{15x}{24} \div 8x^3$

PRACTISING

3. Simplify.

a) $\frac{-15y}{25y^4} \cdot 6y$

b) $14a^2 \div \frac{4a^2}{7}$

c) $\frac{(-2x)^2}{6x^3} \div 12x$

d) $\frac{24a}{18} \cdot \frac{1}{3a^3}$

4. Simplify.

a) $\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right)$

b) $\frac{6a}{9} \div \frac{4a^2}{3}$

c) $\frac{(3y^2)^2}{4y^3} \cdot \frac{20}{y}$

d) $-\frac{15m}{20m^2} \div \frac{3}{14m}$

5. Simplify.

a) $\frac{3x(2x-1)}{7x^2(x-6)} \cdot \frac{14(6-x)}{8x^3}$

b) $\frac{4b^2(2b+1)}{(2b+3)} \div \frac{10(1+2b)}{(b+3)}$

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6. Simplify.

a) $\frac{a^2 + 3a}{25} \cdot \frac{15a^3}{9a + 3a^2}$

b) $\frac{y^4}{5y^2 - 2y^3} \cdot \frac{2y^3 - 5y^2}{y^3}$

c) $\frac{x + 2x^2}{1 - x} \cdot \frac{x^2 - x}{2x^4 + x^3}$

d) $\frac{a^2 - 9}{3a + 9} \cdot \frac{15a^3}{12a^2 - 36a}$

7. Simplify.

a) $\frac{6 + x}{x - 1} \div \frac{4x + 24}{2x - 2x^2}$

b) $\frac{4x^2 + 6x}{10x^3} \div \frac{2x + 3}{15}$

c) $\frac{25a^3}{10a^3 - 20a^2} \div \frac{-a}{4a - 8}$

d) $\frac{6x^4 + x^3}{18x^2 + 6x} \div \frac{-4x^2 - 24x}{10x^3 + 30x^4}$

8. a) Simplify: $\frac{-6x^2}{7x} \div \frac{21}{7x}$

b) Multiply the quotient in part a) by $\frac{9}{7x}$.

9. Simplify: $\frac{5a^2}{10a^3 - 5a^2} \cdot \frac{3a - 6}{a + 2}$

10. Write two rational expressions whose product is $\frac{4}{x^2}$, $x \neq 0, 1$.

11. Write two rational expressions whose quotient is $\frac{-6x}{5}$, $x \neq 0$.

12. Write two rational expressions whose product is $\frac{3x}{x - 2}$, $x \neq \frac{1}{2}, 2$.

13. Write two rational expressions whose quotient is $\frac{y - 3}{9y^2}$, $y \neq 0, 1$.

Closing

14. Create a product of rational expressions with non-permissible values that are not evident in the simplified form. Explain why the simplified form retains the non-permissible values.

Extending

15. a) $\frac{-6x}{15x^3} \div \frac{4(x - 6)}{3x^3(6 - x)}$

b) $\frac{4a(a - 1)(a - 5)}{15a^3(5 - a)} \cdot \frac{-1}{12a^2(a - 1)}$

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4 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study Aid
- See Lesson 4.1, Examples 2 and 3.
- Try Mid-Chapter Review Questions 1 to 3.

Q: Why do you need to state the non-permissible values for a rational expression?

A: In the real number system, division by zero is undefined. For example, you can divide a pizza into 4 equal pieces (each slice is $\frac{1}{4}$ of the pizza), but it does not make sense to divide a pizza into zero equal pieces.

Sometimes there are variables in the denominator of a rational expression. A variable can take any value. If a variable in the denominator ever takes a value that makes the denominator equal to zero, then the rational expression contains division by zero. Division by zero makes an expression undefined. To avoid this problem, state the non-permissible values at the beginning of your solution as restrictions on the variable.

For example, the rational expression

$$\frac{3x}{2x + 1}$$

has a non-permissible value when the denominator is equal to zero.

$$\begin{aligned} 2x + 1 &= 0 \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned}$$

Therefore, $-\frac{1}{2}$ is a non-permissible value.

To ensure that this expression is always defined, use a restriction:

$$\frac{3x}{2x + 1}, x \neq -\frac{1}{2}$$

Study Aid
- See Lesson 4.2, Examples 2 and 3.
- Try Mid-Chapter Review Questions 4 to 6.

Q: Why can you not simplify a rational expression by dividing like terms as shown below?

$$\frac{4x + 5}{4x + 2} = \frac{5}{2}$$

A: Rational expressions are like fractions. To write a fraction in simplest terms, you divide the numerator and the denominator by common factors. In the expression above, $4x$ is not a factor of the numerator or the denominator.

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For example, suppose that you evaluated the expression for $x = 1$ and considered $4(1)$ to be a common factor.

$$\frac{4(1) + 5}{4(1) + 2} \neq \frac{5}{2}$$ This answer is not correct.

If you first evaluate the numerator and the denominator for $x = 1$, following the order of operations, you get the correct answer.

$$\frac{4(1) + 5}{4(1) + 2} = \frac{9}{6}$$

$$= \frac{3(3)}{3(2)}$$

$$= \frac{3}{2}$$ This answer is correct.

You can only simplify a rational expression by dividing common factors, not by dividing common terms.

Q: When is it necessary to analyze the numerator of a rational expression to determine non-permissible values?

A: The only time you would analyze a numerator to determine non-permissible values is when you are dividing two rational expressions. Non-permissible values arise when there is the possibility of dividing by zero. In the example below, there are three different divisions, so there are three polynomials you need to analyze for division by zero.

$$\frac{x + 1}{x + 2} \div \frac{x + 3}{x + 4}$$

The polynomials $x + 2$ and $x + 4$ are denominators of the fractions, so they cannot be equal to zero. Therefore, $x \neq -2$ and $x \neq -4$.

$$\frac{x + 1}{x + 2} \div \frac{x + 3}{x + 4}$$

There is also a third division you need to analyze, because this is a quotient of two rational expressions. The numerator of the second rational expression cannot equal zero, so $x \neq -3$.

Study Aid

- See Lesson 4.3, Examples 2 and 3.
- Try Mid-Chapter Review Questions 7 to 9.

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PRACTISING

Lesson 4.1

1. **a)** Write two different rational expressions that are equivalent to each expression.

i) $\frac{6y}{9y^2}$

ii) $\frac{6}{6 + x}$

iii) $\frac{-12x}{4x^2}$

iv) $\frac{x + 3}{2x - 1}$

b) State the non-permissible values for each rational expression in part a).

2. Which two rational expressions are equivalent? Explain how you know.

$$\frac{3x - 6}{12x + 9}, \frac{2x^2 - 4x}{8x^2 + 6x}, \frac{x - 2}{4x + 3}$$

3. Joanne is riding her new bike along a familiar route. She noticed that she rode the 85 km route 20 min faster than usual.

a) Write a rational expression for her speed.

b) Determine an equivalent rational expression.

c) Determine the non-permissible values, and explain why they are non-permissible.

d) State the restrictions on the expression you wrote for part a).

Lesson 4.2

4. State the non-permissible values of the variable as restrictions, and then simplify.

a) $\frac{2a^2 - 32a}{10a}$

b) $\frac{-4 - 6b}{-6b}$

c) $\frac{15y + 5}{15y}$

d) $\frac{3y^3 - 9y}{2y}$

5. Simplify.

a) $\frac{x^2 - 25}{4x + 20}$

b) $\frac{3x^2 - 15x}{6x - 30}$

c) $\frac{x - 1}{3 - 3x}$

d) $\frac{3a^2 - a^3}{1 - 3a}$

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6. The maximal oxygen consumption, or VO_2 max, is a measure of the circulatory system's potential to transfer oxygen during intense exercise. It is the best measure of cardiac fitness for an endurance athlete. A VO_2 max can only be determined accurately through clinical testing. However, there are several different formulas you can use to estimate your VO_2 max. For example, for an average person of age A, in years,

$$VO_2 \text{ max} = \frac{165\,000 - 750A}{3A + 3450} \text{ mL/min/kg}$$

a) Simplify the rational expression on the right side of this formula.

b) What is the estimated VO_2 max for a person of your age?

c) Does the formula given above have any inadmissible values? Explain.

Lesson 4.3

7. Simplify.

a) $\frac{15x^3}{8x} \cdot \frac{6x}{9x^3}$

b) $5a^3 \div \frac{10a^2}{3}$

8. Simplify.

a) $\frac{3x(3x - 1)}{21(x - 6)} \cdot \frac{14(6 - x)}{2(3x - 1)}$

b) $\frac{(b + 1)}{(2b + 3)} \div \frac{10(1 + b)}{(b + 3)}$

9. Simplify.

a) $\frac{x - 3}{4} \div \frac{1}{4x - 12}$

b) $\frac{2y^2 + 6y}{8y + 2} \cdot \frac{16y^2 - 1}{12y^2}$

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4.4 Adding and Subtracting Rational Expressions

YOU WILL NEED

- calculator

EXPLORE...

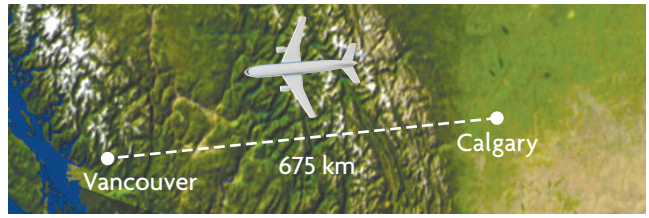
- What number added to $\frac{3}{8}$ will give a sum between $\frac{7}{12}$ and $\frac{11}{16}$?

GOAL

Develop strategies for adding and subtracting rational expressions.

LEARN ABOUT the Math

A jet flew along a straight path from Calgary to Vancouver, and back again, on Monday. It made the same trip on Friday. On Monday, there was no wind. On Friday, there was a constant wind blowing from Vancouver to Calgary at 80 km/h. While travelling in still air, the jet travels at a constant speed.



? Which round trip took less time?

EXAMPLE 1 Selecting a strategy for adding and subtracting rational expressions

Write expressions for the length of time required to fly from Calgary to Vancouver on Monday and Friday. Determine which round trip took less time.

Basil's Solution

Let v represent the jet's airspeed in still air.

Use the relation: time $= \frac{\text{distance}}{\text{speed}}$

Trip on Monday:

Time from Calgary to Vancouver $= \frac{675}{v}$

Time from Vancouver to Calgary $= \frac{675}{v}$

Trip on Friday:

Jet's airspeed from Calgary to Vancouver $= v - 80$

Jet's airspeed from Vancouver to Calgary $= v + 80$

I used the variable v to represent the jet's airspeed in still air, since this is not given.

I determined an expression to represent the time for each leg of the trip at each airspeed.

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$$\text{Time from Calgary to Vancouver} = \frac{675}{v - 80}$$

$$\text{Time from Vancouver to Calgary} = \frac{675}{v + 80}$$

Let T_1 represent the round-trip time on Monday, with no wind:

$$T_1 = \frac{675}{v} + \frac{675}{v}$$

$$T_1 = \frac{1350}{v}$$

I determined each round-trip time by adding the times for both legs of the trip.

I was able to determine the sum for T_1 by adding the numerators, since both rational expressions had a common denominator.

Let T_2 represent the round-trip time on Friday, with wind:

$$T_2 = \frac{675}{v - 80} + \frac{675}{v + 80}$$

$$T_2 = \frac{675(v + 80) + 675(v - 80)}{(v - 80)(v + 80)}$$

$$T_2 = \frac{675v + 54\,000 + 675v - 54\,000}{v^2 + 80v - 80v - 6400}$$

$$T_2 = \frac{1350v}{v^2 - 6400}$$

To determine a sum for T_2, I needed to create an equivalent expression. As with adding rational numbers, when adding rational expressions, the rational expressions being added need to have a common denominator. The common denominator is $(v - 80)(v + 80)$. To maintain equivalence, I must multiply both the numerator and denominator by the same factor.

Creating a common numerator in the expressions for T_1 and T_2:

$$T_1 = \frac{1350}{v} \cdot \frac{v}{v}$$

I noticed that the numerators for T_1 and T_2 are almost the same.

Since the numerators for T_1 and T_2 are now the same, the denominators can be compared directly.

$$T_1 = \frac{1350v}{v^2} \qquad T_2 = \frac{1350v}{v^2 - 6400}$$

The denominator of the expression for T_2 is less, because 6400 is subtracted from v^2.

$$T_1 < T_2$$

The trip on Monday, without wind, took less time.

Since I am dealing with division, the smaller of the two expressions is the expression with the greater denominator, which is T_1.

Reflecting

A. Explain why a greater denominator makes a rational expression smaller.

B. What are the non-permissible values of v, and why are they non-permissible?

C. Compare the strategies for adding and subtracting rational expressions to the strategies for adding and subtracting rational numbers. How are they the same? How are they different?

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APPLY the Math

EXAMPLE 2 Simplifying a sum

Simplify the following sum:

$$\frac{3}{8x^2} + \frac{1}{4x}$$

Akie's Solution

$\frac{3}{8x^2} + \frac{1}{4x}, \quad x \neq 0$

The non-permissible value is $x = 0$.

$= \frac{3}{8x^2} + \frac{1}{4x} \cdot \frac{2x}{2x}$

In this sum, the denominators are different, so I need to determine the lowest common denominator (LCD). Since $8x^2$ is divisible by $8x^2$ and $4x$, it is the LCD. I multiplied the numerator and the denominator of the second rational expression by $2x$ to create an equivalent expression that has $8x^2$ as its denominator.

$= \frac{3}{8x^2} + \frac{2x}{8x^2}$

$= \frac{3 + 2x}{8x^2}, \quad x \neq 0$

As with rational numbers, when rational expressions have common denominators, I can add the numerators to determine the sum of the two expressions.

There are no common factors in the numerator and denominator, so the rational expression cannot be simplified any further.

Your Turn

How will the steps in the solution change if the sum is $\frac{3}{6x^2} + \frac{1}{4x}$?

EXAMPLE 3 Simplifying a difference

Simplify the following difference:

$$\frac{3n}{2n + 1} - \frac{4}{n - 3}$$

Richard's Solution

Non-permissible values:

$2n + 1 = 0 \qquad n - 3 = 0$

$n = -\frac{1}{2} \qquad n = 3$

I determined the non-permissible values by determining the values that make the denominators equal to zero.

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$$\frac{3n}{2n+1} - \frac{4}{n-3}, \quad n \neq -\frac{1}{2}, 3$$

$$= \frac{3n}{2n+1} \cdot \frac{n-3}{n-3} - \frac{4}{n-3} \cdot \frac{2n+1}{2n+1}$$

$$= \frac{3n(n-3) - 4(2n+1)}{(2n+1)(n-3)}$$

$$= \frac{3n^2 - 9n - 8n - 4}{(2n+1)(n-3)}$$

$$= \frac{3n^2 - 17n - 4}{(2n+1)(n-3)}, \quad n \neq -\frac{1}{2}, 3$$

To determine the LCD, I looked at the binomials of each denominator. Since $2n + 1$ and $n - 3$ cannot be factored, they do not have any common factors.

The LCD must be divisible by both denominators, so the LCD is the product $(2n + 1)(n - 3)$.

I created equivalent rational expressions with the LCD.

I expanded the two binomial expressions in the numerator and collected like terms.

I could not factor the numerator, so the numerator and the denominator have no common factors. The expression cannot be simplified further.

Your Turn

Simplify the following difference:

$$\frac{6}{n-3} - \frac{4}{n+2}$$

EXAMPLE 4 Using a factoring strategy to simplify a rational expression

Simplify the following expression:

$$\frac{32}{x^2 - 16} + \frac{4}{x+4}$$

Joyce's Solution

$$\frac{32}{x^2 - 16} + \frac{4}{x+4}$$

$$= \frac{32}{(x+4)(x-4)} + \frac{4}{x+4}$$

$$= \frac{32}{(x+4)(x-4)} + \frac{4}{x+4}, x \neq -4, 4$$

$$= \frac{32}{(x+4)(x-4)} + \frac{4}{(x+4)} \cdot \frac{x-4}{x-4}$$

$$= \frac{32 + 4(x-4)}{(x+4)(x-4)}$$

I noticed that the first rational expression has a difference of squares in the denominator, so I factored it.

The second rational expression cannot be factored further.

I determined the non-permissible values.

The LCD is the product of all the common factors and all the unique factors in the denominators. The common factor is $(x + 4)$ and the unique factor is $(x - 4)$, so the LCD is their product, $(x + 4)(x - 4)$.

I created an equivalent rational expression with this LCD.

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$$= \frac{32 + 4x - 16}{(x + 4)(x - 4)}$$

I expanded the numerator and collected like terms.

$$= \frac{4x + 16}{(x + 4)(x - 4)}$$

$$= \frac{4(x + 4)}{(x + 4)(x - 4)}$$

I factored the numerator to determine if the expression could be simplified further. There is a common factor in the numerator and the denominator of $(x + 4)$, so I simplified.

$$= \frac{4}{x - 4}, \quad x \neq -4, 4$$

I stated the simplified rational expression and restated the restrictions.

Your Turn

Simplify the following expression:

$$\frac{2x}{x^2 - 1} - \frac{4}{x - 1}$$

In Summary

Key Ideas

- The strategies for adding and subtracting rational expressions are the same as the strategies for adding and subtracting rational numbers.
- When rational expressions are added or subtracted, they must have a common denominator.

$$\frac{A}{B} + \frac{C}{D} = \frac{AD + CB}{BD}, B, D \neq 0$$

$$\frac{A}{B} - \frac{C}{D} = \frac{AD - CB}{BD}, B, D \neq 0$$

Need to Know

- To add or subtract rational expressions, follow these steps:
  1. Factor the numerators and denominators of both expressions, if possible.
  2. Determine the lowest common denominator (LCD).
  3. Rewrite each rational expression as an equivalent expression with the LCD as the denominator.
  4. Add or subtract the numerators of the equivalent expressions.
  5. Simplify the rational expression and restate the restrictions on the variable.
- The LCD is the product of all the common factors and all the unique factors of the denominators. The LCD is not always the product of all the denominators.
- The non-permissible values of the variable in the simplified expression are the combination of the non-permissible values of the original expressions.

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CHECK Your Understanding

1. Determine the lowest common denominator for each pair of rational expressions.

a) $\frac{2}{5x^2}$ and $\frac{3y}{10x}$

b) $\frac{-1}{6x^3}$ and $\frac{4y}{9x}$

c) $\frac{2x}{x(x-1)}$ and $\frac{4}{x-1}$

d) $\frac{13}{(x+1)(2x-1)}$ and $\frac{-x}{(2x-1)(x-1)}$

2. Determine the non-permissible values for each pair of rational expressions in question 1.

3. Simplify.

a) $\frac{12x^2}{5x} - \frac{6x^2}{5x}$

b) $\frac{5x+1}{x-1} + \frac{3x-1}{x-1}$

c) $\frac{1}{x^2+2x} - \frac{5x+3}{x^2+2x}$

d) $\frac{3y+7}{y(y-2)} + \frac{4-8y}{(y-2)y}$

PRACTISING

4. Simplify.

a) $\frac{-6a+5}{4a^2} + \frac{3}{6a}$

b) $\frac{3y-7}{6y} - \frac{1}{12y^2}$

c) $\frac{4}{3x} + 5$

d) $\frac{3b}{12} - \frac{5}{4b}$

5. Simplify.

a) $\frac{5x+12}{x(x+6)} - \frac{3}{x}$

b) $\frac{-2}{2y-3} + \frac{1}{4(y+1)}$

c) $\frac{4y-6}{(y-6)(y+6)} - \frac{12-3y}{6(y-6)}$

d) $\frac{-2}{a+2} + \frac{5}{a-2}$

6. Simplify.

a) $\frac{6}{x^2-1} + \frac{1}{x+1}$

b) $\frac{3a}{4a+2} - \frac{3a^2-6}{4a^2-1}$

c) $\frac{9y}{3y+18} + \frac{y-1}{y+6}$

d) $\frac{a}{2a^2-2} - \frac{a}{2a^2}$

7. Simplify.

a) $\frac{7}{2x-6} + \frac{4}{10x-15}$

b) $\frac{5}{2b-2} - \frac{4+b}{2b^2-2b}$

8. Simplify.

a) $\frac{3a}{2a+1} - \frac{3}{4a^2-1}$

b) $\frac{x^2+9}{9-x^2} - \frac{x}{3-x}$

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9. Marlene subtracted two rational expressions and simplified the difference.

$$\frac{2x}{2x+1} - \frac{2x-1}{x+3},\ x \neq -\frac{1}{2}, -3$$

I determined the non-permissible values.

$$= \frac{2x - 2x - 1}{2x + 1 - x + 3}$$

I subtracted the numerators and denominators and simplified the expression.

$$= \frac{-1}{x+4}$$

Marlene made two errors.

a) Identify and explain the errors in Marlene's steps.

b) Correct the errors, and simplify.

10. Charlie paddled his canoe 4 km upstream to get some camping supplies. He then paddled his canoe 4 km downstream, back to camp. The speed of the river was 7 km/h. Assuming that Charlie paddled at a constant speed, write a rational expression for the total time of his trip for supplies.

11. Yvonne invested $3000 in shares of a stock on Tuesday and another $3000 in shares of the same stock on Wednesday. On Wednesday, the price of the stock was $0.15 lower than on Tuesday. Write a rational expression for the difference in the number of shares that Yvonne purchased on Tuesday and Wednesday.

Closing

12. Explain how to determine the LCD for a sum or difference of rational expressions. Use examples in your explanation.

Extending

13. Simplify.

$$\left(\frac{x^2 - x - 6}{2x - 3}\right)\left(\frac{3}{x-3} - \frac{9x}{x^2 - 9}\right)$$

14. Simplify.

$$\left(\frac{1}{a^2} - \frac{1}{9}\right) \div \left(\frac{1}{3a}\right)$$

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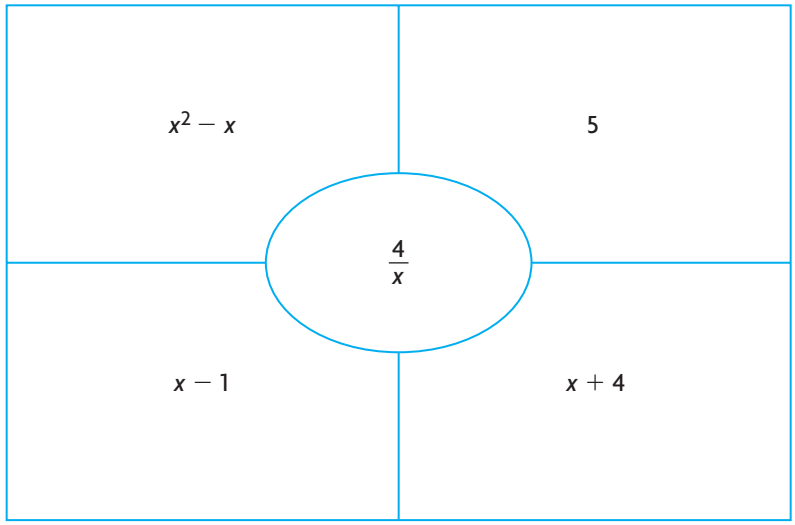
Applying Problem-Solving Strategies

Exploring Rational Expressions



The Puzzle

A. Create two rational expressions using the polynomials in the four corners of the rectangle.

B. Add, subtract, multiply, or divide your two rational expressions. Then simplify to determine if your result is $\frac{4}{x}$.

C. If your result is not $\frac{4}{x}$, repeat parts A and B until it is.

The Strategy

D. Describe the strategy you used to solve the puzzle.

Variation

E. Create your own puzzle, similar to the one above, by writing four different polynomial expressions in each corner. What rational expression will you place in the oval?

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YOU WILL NEED

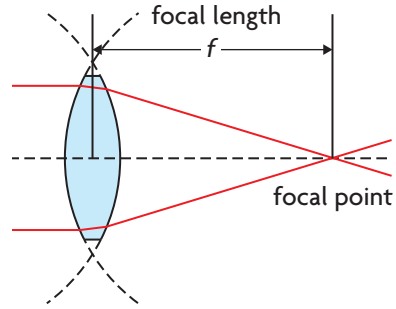
- magnifying glass or reading glasses
- measuring tape or metre stick

History | Connection

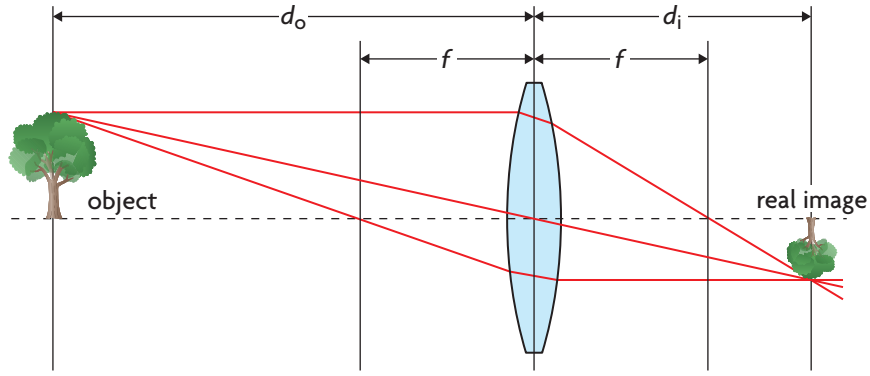
The Thin Lens Formula

Humans have used optical lenses for thousands of years. The Nimrud lens was discovered in Iraq in 1850 and is over 3000 years old.

The property of bending or refracting light toward a common point is what makes a convex lens so useful. This common point is called the *focal point*. The distance from the focal point to the convex lens is called the *focal length*, f, of the lens. A convex lens has a higher magnification (optical power) when the focal point is closer to the lens.



When light from an object passes through a simple lens, the light is refracted at each surface of the lens. An upside-down image can be seen on the other side of the convex lens.



To determine the focal length, f, of a convex lens, measure the distance to the object (d_o) and the distance to the image (d_i). Then use the thin lens formula:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

A. Find a regular thin convex lens, in a magnifying glass or a pair of reading glasses, that you can use. Position an object 100 cm from the lens (d_o). With a classmate, design an experiment to determine the distance from your lens to the image of the object (d_i). Conduct your experiment.

B. Use the thin lens formula to determine the focal length of your lens.

C. Rewrite the thin lens formula so that it can be used to determine f.

D. How can you verify that your formula can be used to determine f?

E. Determine the non-permissible values for your formula. Explain why they are non-permissible.

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4.5 Solving Rational Equations

GOAL

Solve rational equations, and verify their solutions.

YOU WILL NEED

- calculator

EXPLORE...

- When I divide 6 by a number and then add the result to its reciprocal, I get 2.5. What is my number?

LEARN ABOUT the Math

Salt water is flowing into a large tank that contains pure water. The concentration of salt in the tank, c, in grams per litre (g/L), at time t, in minutes, is given by the formula:

$$c = \frac{10t}{25 + t}$$

? When does the salt concentration in the tank reach 3.75 g/L?

EXAMPLE 1 Solving a problem that involves a rational equation

Determine the time when the salt concentration in the tank reaches 3.75 g/L.

Lukas's Solution

If the salt concentration is 3.75, $c = 3.75$.

$$3.75 = \frac{10t}{25 + t}, \quad t \geq 0$$

I substituted 3.75 for c in this **rational equation**. The non-permissible value of t is -25. However, since t measures the time since the salt water started flowing, a negative time makes no sense. Therefore, $t \geq 0$.

$$(25 + t)(3.75) = (25 + t)\frac{10t}{(25 + t)}$$

I multiplied both sides of the equation by the LCD, $(25 + t)$, and simplified.

$$93.75 + 3.75t = 10t$$
$$93.75 = 6.25t$$
$$15 = t$$

I solved for t.

It takes 15 min for the salt concentration to reach 3.75 g/L.

Reflecting

A. What is happening to the concentration of salt in the water as time passes? Explain.

B. How could you verify that Lukas's solution is correct?

C. Explain, using an example, why Lukas multiplied both sides of the equation by the LCD.

rational equation

An equation that involves one or more rational expressions; for example:

$$\frac{5}{x} = \frac{4}{x + 2}$$

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APPLY the Math

EXAMPLE 2 Solving a rational equation with an extraneous root

Solve the following equation for x:

$$\frac{18}{x^2 - 3x} = \frac{6}{x - 3} - \frac{5}{x}$$

Marlene's Solution

$$\frac{18}{x(x - 3)} = \frac{6}{x - 3} - \frac{5}{x}, \quad x \neq 0, 3$$

I factored first to determine the non-permissible values and the LCD.

$$x(x - 3) \cdot \frac{18}{x(x - 3)} = x(x - 3) \cdot \frac{6}{(x - 3)} - \frac{5}{x} \cdot x(x - 3)$$

The LCD is $x(x - 3)$ because it is the product of the common and unique factors of the denominators of the expressions.

Because I am working with an equation, I can multiply all the terms by the LCD to eliminate the fractions.

$$\begin{aligned} 18 &= 6x - 5(x - 3) \\ 18 &= 6x - 5x + 15 \\ 3 &= x \end{aligned}$$

I simplified, then solved for x.

The root is $x = 3$, but this is a non-permissible value.
Therefore, $x = 3$ is an **extraneous root**.

There is no solution to this equation.

Your Turn

Choose one of the following equations, and solve it.

$$\frac{x}{2} - \frac{2x + 5}{4} = \frac{4}{3} \quad \text{or} \quad \frac{2}{a + 2} - \frac{a^2 + 4}{a^2 - 4} = \frac{a}{2 - a}$$

Communication *Tip*

An *extraneous* root is a solution to an equation that is not permissible in the original equation. An *inadmissible* solution is a solution that is permissible in the original equation but is not valid in the context of the problem.

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EXAMPLE 3 Solving a rational equation with an inadmissible solution

When they work together, Stuart and Lucy can deliver flyers to all the homes in their neighbourhood in 42 min. When Lucy works alone, she can deliver the flyers in 13 min less time than Stuart can when he works alone. When Stuart works alone, how long does he take to deliver the flyers?

James's Solution

Let y represent the time that Stuart takes to deliver the flyers when he works alone.

Then $(y - 13)$ represents the time that Lucy takes to deliver the flyers when she works alone.

I chose a variable to represent Stuart's time and used it to write an expression for Lucy's time.

The fraction of deliveries made in 1 min

- by Stuart working alone: $\frac{1}{y}$
- by Lucy working alone: $\frac{1}{y - 13}, y > 13$
- by Stuart and Lucy working together: $\frac{1}{42}$

I compared the rates at which Stuart and Lucy work. For example, if Stuart took 80 min to deliver all the flyers, he would deliver $\frac{1}{80}$ of the flyers per minute.

Because Stuart takes longer than Lucy to deliver the flyers, and because the denominator cannot equal zero, $y > 13$.

I wrote an equation that relates these three rates.

$$\frac{1}{y} + \frac{1}{y - 13} = \frac{1}{42}$$

$$42y(y - 13)\left(\frac{1}{y}\right) + 42y(y - 13)\left(\frac{1}{y - 13}\right) = 42y(y - 13)\left(\frac{1}{42}\right)$$

$$\frac{42y(y - 13)}{y} + \frac{42y(y - 13)}{y - 13} = \frac{42y(y - 13)}{42}$$

There are no common factors in the denominators, so the LCD is the product of the three denominators: $42y(y - 13)$

I multiplied each term by the LCD and then simplified.

$$\frac{42\color{magenta}{y}(y - 13)}{\color{magenta}{y}} + \frac{42y(\color{magenta}{y - 13})}{\color{magenta}{y - 13}} = \frac{\color{magenta}{42}y(y - 13)}{\color{magenta}{42}}$$

$$42(y - 13) + 42y = y(y - 13)$$

$$42y - 546 + 42y = y^2 - 13y$$

$$0 = y^2 - 97y + 546$$

I simplified and solved for y.

$$0 = (y - 91)(y - 6)$$

$$y = 91 \text{ or } y = 6$$

I checked for inadmissible solutions. Since Lucy takes 13 min less than Stuart, the time that Stuart takes must be greater than 13 min.

Since $y > 13$, $y = 6$ is an inadmissible solution.
It takes Stuart 91 min to deliver the flyers when he works alone.

Your Turn

Verify that James's solution is correct.

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EXAMPLE 4 Using a rational equation to model and solve a problem

Rima bought a case of concert T-shirts for $450. She kept two T-shirts for herself and sold the rest for $560, making a profit of $10 on each T-shirt. How many T-shirts were in the case?

Al-Karim's Solution

Let x represent the number of T-shirts in the case:

Buying price per T-shirt $= \frac{450}{x}$

Rima paid $450 for x T-shirts, so each T-shirt cost $\frac{450}{x}$.

Selling price per T-shirt $= \frac{560}{x - 2}$

She kept two T-shirts for herself, which left $(x - 2)$ T-shirts for her to sell. Rima sold $(x - 2)$ T-shirts for $560, so she charged $\frac{560}{x - 2}$ for each T-shirt.

Profit per T-shirt = Selling price − Buying price

Her profit is the difference between the selling price and buying price.

Profit per T-shirt $= \frac{560}{x - 2} - \frac{450}{x}, \quad x > 2, \quad x \in \mathrm{W}$

The non-permissible values of the variable in the expressions are $x = 0$ and $x = 2$. To make a profit, however, there had to be more than two T-shirts in the case. Therefore, the restriction on the variable is $x > 2$. The variable is also restricted to the set of whole numbers, since Rima cannot have partial T-shirts.

$$10 = \frac{560}{x - 2} - \frac{450}{x}$$

I wrote a rational equation by setting the profit expression equal to 10, since Rima made a profit of $10 on each T-shirt.

$$10x(x - 2) = \frac{560x(x - 2)}{x - 2} - \frac{450x(x - 2)}{x}$$

I multiplied both sides of the equation by the LCD, $x(x - 2)$.

$$10x(x - 2) = \frac{560x(x - 2)}{x - 2} - \frac{450x(x - 2)}{x}$$

I simplified the terms on the left side of the equation to create a quadratic equation in standard form.

$$10x(x - 2) = 560x - 450(x - 2)$$
$$10x^2 - 20x = 560x - 450x + 900$$
$$10x^2 - 130x - 900 = 0$$
$$10(x^2 - 13x - 90) = 0$$
$$10(x - 18)(x + 5) = 0$$

I solved for x by factoring.

$$x = 18 \text{ or } x = -5$$

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Since $x > 2$, $x = -5$ is an inadmissible solution.

There could not be -5 T-shirts in the case.

Verify for $x = 18$:

$$10 = \frac{560}{x - 2} - \frac{450}{x}$$

LS	RS
10	$\frac{560}{18 - 2} - \frac{450}{18}$ $\frac{560}{16} - \frac{450}{18}$ $\frac{70}{2} - \frac{50}{2}$ $\frac{20}{2}$ 10

There were 18 T-shirts in the case.

Your Turn

Estimate the number of T-shirts in the case if the profit per T-shirt increased to \$12. Verify your estimate.

In Summary

Key Ideas

- You can solve a rational equation algebraically by multiplying each term in the equation by the lowest common denominator (LCD) to eliminate fractions from the equation. Then solve the resulting linear or quadratic equation.
- If a root of a rational equation is a non-permissible value of the rational expressions in the original equation, it is an extraneous root and must be dismissed as a valid solution.

Need to Know

- Each solution to a rational equation can be verified by substituting it into the original equation. If the left side equals the right side, then it is a valid solution to a rational equation in a non-contextual problem.
- When solving a contextual problem, it is important to check for solutions that are valid solutions to the equation but are inadmissible in the context of the problem.

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CHECK Your Understanding

1. State the non-permissible values, and then solve each rational equation.

a) $\frac{1}{x} + \frac{1}{3x} = \frac{1}{6}$

b) $\frac{2y - 6}{4y - 8} = 1$

c) $\frac{1}{x} - \frac{1}{x + 1} = \frac{1}{6}$

d) $\frac{3}{x - 5} = \frac{10}{5x - 25}$

2. Define a variable and write a rational expression to describe each situation.

a) The time it takes to travel 15 km when the speed has been decreased by 5 km/h.

b) The unit cost when the total cost is $4000 and the number of items has increased by 5.

c) The speed of a car that travelled 50 km in twice the time.

d) The fraction of a job completed by Shannon in 1 min if it takes her 5 min longer than Brad to complete the job.

3. Determine the non-permissible values of the variable in each part of question 2. Then state the restrictions on the variable.

4. Are $x = 3$ and $x = -2$ roots of the following equation?

$$\frac{2}{x} = \frac{x - 1}{3}$$

Explain how you know.

PRACTISING

5. State the non-permissible values of the variable, and then solve each rational equation. Verify your solution.

a) $\frac{x + 3}{x - 1} = 0$

b) $\frac{x + 3}{x - 1} = 2$

c) $\frac{x + 3}{x - 1} = 2x + 1$

d) $\frac{3}{3x + 2} = \frac{6}{5x}$

6. State the non-permissible values of the variable, and then solve each rational equation. Verify your solution.

a) $\frac{1}{x - 3} - \frac{6}{x^2 - 9} = \frac{1}{5}$

b) $\frac{z}{z - 1} - \frac{1}{z - 1} = -4$

c) $\frac{5}{x + 3} = \frac{2 - 4x}{4x} + 2$

d) $\frac{2}{m - 3} + \frac{1}{m} = \frac{6}{m^2 - 3m}$

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7. The Greek mathematician Pythagoras is credited with discovering the golden rectangle. This is considered to be the rectangle with the most visually appealing dimensions. In a golden rectangle, the length, l, and the width, w, are related by the following equation:

$$\frac{l}{w} = \frac{w}{l - w}$$

A billboard with a length of 15 m is being designed. If the designers want the billboard to be a golden rectangle, what must its width be?

8. The Turtledove Chocolate factory has two chocolate machines. Machine A takes t min to fill a case with chocolates, and machine B takes $t + 10$ min to fill a case. Working together, the two machines take 15 min to fill a case. Approximately how long does each machine take to fill a case?

9. Michael is a bicycle commuter. He has observed that he reduces the time for his 9 km commute by 15 min when he increases his average speed by 3 km/h. What is Michael's faster speed?

a) Define a variable and write an expression for Michael's slower time.
b) Use the same variable to write an expression for his faster time.
c) Write a rational equation to represent Michael's faster speed.
d) Determine the non-permissible values of the variable and the restrictions on the variable.
e) Solve your equation to determine Michael's faster speed.

10. When working alone, it takes Susan 3 h to paint a room and Jacqueline 2 h to paint a room that is the same size. How long would it take them to paint a room if they worked together?

11. When working together, brothers Jonah and Caleb can complete their farm chores in 80 min. Working independently, it takes Jonah 36 min longer than his brother to complete the chores. How long does it take Caleb to complete the chores when working alone?

12. It takes Rahj 9 h longer than Enzo to build an outdoor patio. If they work together, they can build a patio in 20 h. How long does it take each person to build a patio when working alone?

13. Tara is training for a marathon. Her personal best (PB) time to run 10 km is 36 min. At 4 km in a 10 km run, Tara looks at her wristwatch computer to see if she is on pace to beat her PB. The computer indicates that she needs to increase her speed by 3 km/h to finish in 36 min. How fast does Tara need to run the remaining 6 km to finish in 36 min?

14. Lydia frequently drives 189 km to visit friends in Canmore, Alberta. She has noticed that she saves 20 min if she travels 24 km/h faster than her average speed. What is her average speed?

The town of Canmore, Alberta, sits at the base of the Rocky Mountains, near Banff National Park.

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15. Sarah owns a bike store. She ordered $60 000 worth of identical bicycles from a manufacturer. She decided to give three of the bicycles to her three children, and then she sold the remaining bicycles for $70 500. Sarah made a profit of $300 per bicycle sold. How many bicycles did she order?

16. Sarah also ordered two different types of tires. She spent $1200 on high-performance tires and $3000 on regular tires. Each of the regular tires cost $3 less than a high-performance tire. She ordered 330 tires in total. What was the cost of each high-performance tire?

17. Lee, a math teacher in Edmonton, organized a field trip to Calaway Park so that his students could study the mathematics of roller coasters. The cost of the trip was $5400, which was divided equally among all the students. At the last moment, 5 students were unable to go, so each of the other students had to pay an additional $15. How many students went on the field trip?

The Métis traditionally used dried wolf willow seeds to make necklaces and bracelets. When dry, wolf willow seeds appear striped.

18. Cheyenne purchased $75 worth of supplies to make wolf willow bead necklaces. She plans to keep three necklaces for her family and sell the remaining necklaces at the upcoming Pow Wow for a total of $250. If Cheyenne intends to make a profit of $12.50 per necklace, how many necklaces does she need to make?

Closing

19. Design a problem that involves a rational equation and has a whole-number solution. (You could use one of the Practising questions as a model.) Exchange problems with a classmate, and solve your classmate's problem. Explain the strategy you used to solve the problem.

Extending

20. Theresa takes 3 min less than Pietr to load a truck when they work alone. One morning, Theresa spent 6 min loading a truck before going on her lunch break. Pietr decided to finish loading the truck and completed the job in 8 min. How long would it have taken Pietr to load the truck on his own?

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Math in Action

The Harmonic Mean

The arithmetic mean is the mean that you are most familiar with. It is determined by adding two or more numbers and then dividing by the number of numbers. The arithmetic mean is used for calculating most everyday averages, but it does not always work.

- Read the following problem, and estimate the solution:

  Niobe has ridden the first half of a 100 km bicycle trip at 20 km/h. At what speed does she need to ride the second half of the trip to have an average speed of 25 km/h?

- Test your estimate by determining the average speed:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

- Test other estimates until you come up with a speed that solves the problem.

The harmonic mean can be used to solve the problem.

If a represents Niobe's speed during the first half of the trip, and b represents her speed during the second half of the trip, her harmonic mean speed can be represented by the formula:

$$H = \frac{2ab}{a + b}$$

- Use this formula for the harmonic mean to determine Niobe's speed for the second half of the trip.

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4 Chapter Self-Test

1. Simplify.

a) $\frac{-38a}{16a^2}$ **b)** $\frac{15x(2-x)}{x-2}$ **c)** $\frac{12x^2+24x}{6x^2}$ **d)** $\frac{b-1}{15-15b}$

2. Simplify.

a) $\frac{12(2b-1)}{2b} \cdot \frac{1}{6b(2b-1)}$

b) $\frac{3x^2-3x}{10} \div \frac{x-1}{5x}$

c) $\frac{3-x}{x} \div \frac{9-x^2}{x^3}$

d) $\frac{12a}{a-3} \cdot \frac{2a-6}{-6a}$

3. Simplify.

a) $\frac{4}{b} - \frac{b+6}{2b}$

b) $\frac{-6x^3}{2x(x+1)} + 3x$

c) $\frac{6x+5}{1-4x^2} - \frac{15}{2+4x}$

d) $\frac{3}{x-6} - \frac{1}{2x-12}$

4. Solve.

a) $\frac{3x+2}{x-2} + 1 = -5$

b) $\frac{3}{a} - 6 = \frac{5a}{2a^2}$

c) $\frac{a}{a-1} - \frac{2}{1-a} = 7$

d) $5 - \frac{2x-1}{x+1} = -\frac{2x-1}{x+1}$

5. A private airplane flies at a speed of 220 km/h in still air. On a trip from Red Deer to Fort McMurray, the airplane flies into a headwind. Then it returns to Red Deer with a tailwind. The total distance for the trip is 550 km, and the total time is 5 h 3 min. Determine the speed of the wind.

6. A retailer imports a crate of sunglasses for $4500. Unfortunately, 10 pairs of sunglasses are defective and cannot be sold. The retailer sells the remaining sunglasses for $8900, realizing a profit of $5 per pair. How many pairs of sunglasses did he sell?

7. Kathryn burns 125 Cal per hour more when she is running compared to when she is cycling. In one workout, she burned 690 Cal on her bicycle and then 1260 Cal on her run. Her total workout lasted 3 h. How many Calories per hour does Kathryn burn when she is cycling?

8. Two brothers, John and Colin, are responsible for shovelling their family's driveway and sidewalk after a snowfall. John takes x min to complete the shovelling on his own, and Colin takes 25 min longer. If they work together, it takes them half an hour to complete the shovelling. How long does it take each of them when they work alone?

WHAT DO You Think Now? Revisit **What Do You Think?** on page 215. How have your answers and explanations changed?

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4 Chapter Review

FREQUENTLY ASKED Questions

Q: Why is factoring important when you simplify a rational expression?

A: Factoring can play up to three different roles when you simplify a rational expression:

- Factoring can help you determine the non-permissible values.

$\frac{1}{3x^2 - 12x}$ — To determine the non-permissible values, determine the values that make the denominator equal to zero. This is easiest to do when the expression is factored.

$= \frac{1}{3x(x - 4)}, x \neq 0, 4$ — The non-permissible values are $x = 0$ and $x = 4$.

- Factoring can help you simplify a rational expression or the product or quotient of rational expressions.

$\frac{x - 4}{3x^2 - 12x}$ — To simplify this rational expression, you need to factor it first.

$= \frac{(x - 4)}{3x(x - 4)}, \quad x \neq 0, 4$

$= \frac{(x - 4)}{3x(x - 4)}$ — Once it is factored, you can see that the numerator and denominator have the common factor $(x - 4)$.

$= \frac{1}{3x}, \quad x \neq 0, 4$ — This rational expression cannot be simplified any further.

- Factoring can help you determine the lowest common denominator (LCD) when you are adding or subtracting rational expressions.

$\frac{1}{3x^2 - 12x} - \frac{1}{x^2 - 16}$ — To determine the LCD, you need to factor all the denominators.

$= \frac{1}{3x(x - 4)} - \frac{1}{(x - 4)(x + 4)}, x \neq -4, 0, 4$ — The LCD is the product of all the common factors and all the unique factors of the denominators. Here, $(x - 4)$ is a common factor, and $3x$ and $(x + 4)$ are the unique factors. The LCD is $3x(x - 4)(x + 4)$.

Study *Aid*

- See Lesson 4.4, Examples 3 and 4.
- Try Chapter Review Questions 7 to 9.

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$$\frac{1}{3x(x-4)} \cdot \frac{x+4}{x+4} - \frac{1}{(x-4)(x+4)} \cdot \frac{3x}{3x}$$
$$= \frac{(x+4)}{3x(x-4)(x+4)} - \frac{3x}{3x(x-4)(x+4)}$$
$$= \frac{x+4-3x}{3x(x-4)(x+4)}$$
$$= \frac{4-2x}{3x(x-4)(x+4)}, x = -4, 0, 4$$

Multiply the numerator and the denominator of each term by the factors of the LCD that are not present in the denominator of the term. This is the same method you use when adding or subtracting fractions.

Study Aid

- See Lesson 4.5, Examples 2 and 4.
- Try Chapter Review Questions 10 to 12.

Q: How do you solve and verify the roots of a rational equation?

A: You can solve a rational equation algebraically by multiplying each term in the equation by the lowest common denominator (LCD) and then solving the resulting polynomial equation. For example, solve:

$$\frac{3x-8}{2x-1} = \frac{x-4}{x+1}, x \neq -1, \frac{1}{2}$$

$$\frac{3x-8}{2x-1} \cdot (2x-1)(x+1) = \frac{x-4}{x+1} \cdot (2x-1)(x+1)$$
$$(3x-8)(x+1) = (x-4)(2x-1)$$
$$3x^2 + 3x - 8x - 8 = 2x^2 - x - 8x + 4$$
$$0 = -x^2 - 4x + 12$$
$$0 = -(x+6)(x-2)$$
$$x = -6 \text{ or } x = 2$$

Multiply each term in the equation by the LCD, which in this case is $(2x-1)(x+1)$.

You can verify the roots by substituting them into the original equation. For example, to verify $x = -6$, substitute -6 into each side of the original equation.

LS	RS
$\frac{3(-6)-8}{2(-6)-1}$	$\frac{(-6)-4}{(-6)+1}$
$\frac{-18-8}{-12-1}$	$\frac{-10}{-5}$
$\frac{-26}{-13}$	2
2	

The left side and the right side are equal, so $x = -6$ is a root of the equation.

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PRACTISING

Lesson 4.1

1. Write two different rational expressions that are equivalent to each expression below.

a) $\frac{12}{8x + 4}$

b) $\frac{x - 3}{x + 3}$

c) $\frac{8a^2}{2a - 6}$

d) $\frac{16m^3 - 12m^2}{6m^4}$

2. The area of a rectangular property is 320 m^2. The length is 6 m longer than the width, w. Write a rational expression for the length of the property in terms of w. Then explain why some values of w are non-permissible.

Lesson 4.2

3. Simplify.

a) $\frac{6n}{14n^3}$

b) $\frac{-15y^4}{20y(y - 2)}$

c) $\frac{36x^3}{20x^3(2x - 1)}$

d) $\frac{4(a - 2)(a + 1)}{12a(a + 1)}$

4. Simplify.

a) $\frac{x - 6}{30 - 5x}$

b) $\frac{18a^2}{12a - 24a^2}$

c) $\frac{36y^2 - 1}{12y^2 - 2y}$

d) $\frac{7a^3 - 28a}{15a + 30}$

Lesson 4.3

5. Simplify.

a) $\frac{1}{4x^2} \cdot \frac{22x}{5}$

b) $\left(\frac{15a}{3a - 6}\right)\left(\frac{2 - a}{10a^2}\right)$

c) $\frac{m^2 + 2m}{m^2 - 4} \cdot \frac{12m - 24}{16m}$

d) $\left(\frac{36y^2 - 1}{12y^2 - 2y}\right)\left(\frac{15}{30y + 5}\right)$

6. Simplify.

a) $\frac{6x^2}{15x^3} \div \frac{4x(x - 1)}{20x^2}$

b) $\frac{1 - y^2}{8 + 8y} \div \frac{3y^2 - 3}{2y + 2}$

c) $\frac{9 - x^2}{x^2} \div \frac{3}{x + 3}$

d) $\frac{-7a}{7a - 14} \div \frac{14}{a - 2}$

Lesson 4.4

7. Determine the lowest common denominator for each pair of rational expressions.

a) $\frac{1}{8x^2}$ and $\frac{y}{12x^3}$

b) $\frac{5}{3x - 6}$ and $\frac{6x}{6x - 12}$

c) $\frac{4a}{a^2 - 64}$ and $\frac{6}{5a - 40}$

d) $\frac{a + 3}{3 - a}$ and $\frac{16a}{2a - 6}$

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8. Simplify.

a) $\frac{3}{x+1} - \frac{2}{3x}$

b) $\frac{1}{10x} - \frac{4-x}{6x}$

c) $3b - \frac{2}{1+3b}$

d) $\frac{1}{x+2} + \frac{3}{x-2}$

9. Simplify.

a) $\frac{1}{y^2+y} + \frac{2}{1-y^2}$

b) $a + \frac{a-3}{a-1}$

c) $\frac{5}{2x-2} - \frac{x+4}{2x^2-2x}$

d) $\frac{5}{a-4} - \frac{a}{a^2-16}$

Lesson 4.5

10. Solve.

a) $\frac{5x-4}{3x-1} = \frac{3}{2}$

b) $\frac{12}{10+y} + \frac{12}{10-y} = \frac{5}{2}$

c) $\frac{1}{2x+8} = \frac{16}{2x-32}$

d) $\frac{a}{3a-9} = \frac{54}{3a^2-27}$

11. Tayla purchased a large box of comic books for $300. She gave 15 of the comic books to her brother. Then she sold the rest of the comic books on an Internet website for $330, making a profit of $1.50 on each.

a) How many comic books were in the box?

b) What did Tayla pay for each comic book?

12. Dallas Fiddler-Boyer is a Métis fiddler from Saskatoon. He is planning a trip to Batoche, Saskatchewan, where he will perform old-time and traditional music. Dallas plans to drive north along Highway 11 for 20 km, before turning onto country roads for the remaining 40 km. Dallas can travel 20 km/h faster on the highway than he can on country roads. If he plans to make the entire trip in 42 min, at what speed must Dallas drive on the highway?

Dallas Fiddler-Boyer performed during the opening ceremonies at the 2010 Olympics, representing the Métis Nation of Canada.

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4 Chapter Task

The Rational Expressions Dice Game

You will design, build, and play a dice game that will help you reinforce your skills with rational expressions.

YOU WILL NEED

- scissors
- clear tape
- calculator
- cube nets

Designing and Building Your Dice

1. Create 12 different rational expressions that satisfy the following conditions:
   - Each expression uses one variable.
   - Each expression may contain monomials, binomials, or both.
2. Write all 12 rational expressions on two cube nets. Write a different rational expression on each face.
3. Write each operation $(+, -, \times, \div)$ on a different face of the third cube net. Write "wild ops" on the remaining two faces.
4. Fold all three cube nets into cubes, and secure the edges. These will act as your dice.

Playing the Game

5. Players take turns rolling the three dice.
   - All the players use the operation that is rolled with the operations die to combine the two rational expressions on the other two dice. If the simplification involves eliminating common factors, then the player who rolled the dice scores 1 point for each factor eliminated.
   - If "wild ops" is rolled with the operations die, then the player who rolled the dice chooses an operation. If the simplification involves eliminating common factors, then the player who rolled the dice scores 2 points for each factor eliminated.
   - If the simplification does not involve eliminating factors, no points are scored.
6. The game ends after three rounds. The player with the highest score wins.

? How can you design a high-scoring game?

A. What form of the polynomials in the rational expressions will make completing an operation and counting your score as easy as possible?

B. Why should the 12 rational expressions have common factors?

C. Play the game.

D. List some strategies you used to maximize your score.

Task Checklist

- ✔ Did you justify your dice design?
- ✔ Was your explanation of your strategies clear?
- ✔ Did you use appropriate mathematical terms?

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4 Project Connection

Carrying Out Your Research

As you continue with your project, you will need to conduct research and collect data. The strategies that follow will help you collect data.

Considering the Type of Data You Need

Both primary data and secondary data have their pros and cons. Primary data provides specific information about your research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analyzed in less time. However, because the data was gathered for other purposes, you may need to sift through the data to find what you are looking for.

The type of data you choose can depend on many factors, including the research question, your skills, and available time and resources.

Assessing the Reliability of Sources

When collecting primary data, you must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys or interviews, the questionnaires must be designed to avoid bias.
- For experiments or studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When obtaining secondary data, you must ensure that the source of your data is reliable.

- If the data is from a report, determine what the authors' credentials are, how up-to-date the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help you decide how reliable the data is.

Accessing Resources

Your school or community librarian can help you locate relevant sources, as can the librarians of local community colleges or universities. Other people, such as teachers, your parents or guardians, local professionals, and Elders, may also have valuable input. (Be sure to respect local community protocols when seeking involvement of Elders or Knowledge Keepers.) Make a list of people who might be able to help you obtain the information you need, and then identify how you might contact each person on your list.

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Project Example | Carrying out your research

Walther's research question was, "Which pests or invasive species cause the greatest damage to our natural resources in the Western provinces and territories, and what can be done to control these threats?"

Walther's Search

Since my question may involve a large number of different species of plants, animals, and insects over a wide area, I decided to rely on secondary data. I started my search using the Internet. I did a search for "pests and invasive species in Canada" and found some websites. The first I visited was a government website that had lots of useful information.

Some other websites I visited, including the Canadian Wildlife Federation, the Canadian Aquatic Invasive Species Network, and the Canadian Botanical Conservation Network, were also helpful. In addition, I came across some print resources to look at. I plan to ask the librarian to help me locate some of these resources.

I feel confident that these sources are reliable and credible. I will continue looking for additional data that may be of use.

Your Turn

A. Decide if you will use primary data, secondary data, or both. Explain how you made your decision.

B. Make a plan you can follow to collect your data.

C. Carry out your plan to collect your data. Make sure to record your successful searches, so you can easily access these sources at a later time. You should also record detailed information about your sources, so you can cite them in your report. See your teacher for the preferred format for endnotes, footnotes, or in-text citations.

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